Topic Study Group 35

TSG-35: Knowledge and Practice of Mathematics Teacher Educator

TSG-35 Working team[1]

1. Introduction of the Background

The last decades of research in mathematics education are best summarized by Anna Sfard's survey team at ICME-10 (2004) as "the era of the teacher" due to researchers' uncontested focus on teachers. Such attention is also represented in the launching in 1998 of an international journal dedicated to mathematics teachers' education, the Journal of Mathematics Teacher Education. Questions about what teachers need to know and be able to do, as well as how they develop their knowledge, skills, and beliefs have become central to the mathematics education research literature.

More recently, there is also growing attention on mathematics teacher educators (MTEs), that is, those who educate mathematics teachers, who design and implement opportunities for mathematics teacher education and development (MaTED). The goal of the TSG-35 at ICME-14 is two-fold: to collect information about mathematics teacher educators working in a variety of MaTED programs around the world and understand their contexts and cultures; and to discuss growing research about mathematics teacher educators, their knowledge, practice and beliefs.

The wording "mathematics teacher educator" (MTE) in some sense suggests a focus on academics only. This may be true for those countries/regions where MaTED is mainly at universities. But there are countries/regions where MaTED takes place within the instruction system or in teacher education institutes that are independent of universities. The recently launched ICMI Study 25 (co-chaired by Potari and Borko) focuses on the idea of mathematics teachers learning through collaboration in schools or larger communities, drawing on an ICME-13 survey team by Robutti et al. Collaborative groups may be teams, communities, schools and other educational

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institutions, professional development courses, local or national networks. This means that mathematics teacher educators can be working in formal or informal groupings, in either face-to-face or distance settings. They can be facilitators such as trainers, coaches, or mentors.

2. Paper Invitation and Submission for TSG-35

Given the variety of ways in which mathematics teacher educators can work, and the different settings in which they can operate, we invite papers that address the growing need to further understand these professionals.

Some questions to be answered might be:

1. Who are mathematics teacher educators?
2. What do we know about their knowledge, practice, and beliefs?
3. What is their work and under what conditions do they operate?
4. What framework should we adopt to illustrate different aspects of their knowledge?
5. What and how do the different avenues/contexts contribute to the growth in their practice?
6. What counts as experience and what difference does it make in their practice?
7. In what ways do mathematics teacher educator and teacher's knowledge and beliefs come into play in teacher education contexts?

To answer similar questions several teacher educators from different parts of the world met virtually on the occasion of the ICME-14 in shanghai. The list of papers presented can be found in Tab. 1 (on the next page).

3. A Brief Description of the Contributions

To address the first question, Goos and Marshman[1] took a sociocultural perspective on MTEs learning as identity formation. Brief case studies of MTEs who participated in two Australian research studies were illustrated.

MTEs' competence including knowledge is concerned in many ways under different contexts. Alacaci et al.[20] compared two frameworks of competencies for MTEs in mathematics and in technology, aiming to contribute to the discourse on the nature of teacher educator competency frameworks, identify areas of variances, and suggest possible reasons as well as inherent tensions. Huang[21] examined pedagogical practices of MTEs who provide online professional education programs for prospective and practicing mathematics teachers to unpack MTEs' knowledge. Three MTEs' knowledge structure was analyzed using teacher educator knowledge tetrahedron. Cross-case analysis revealed knowledge that was specific for MTEs' decisions about online mathematics teacher educating.

Focused on the practice, Kumar[2] made an analysis of math teacher educator's practice using vignettes illustrate the way different knowledge needs to be integrated in practice of teacher educator in designing and facilitating the tasks and discussion using them. The analysis revealed that though the dialogic approach in workshops

Tab. 1. The list of papers presented

Paper and author(s):
Session 1
[1] Boundary crossing and mathematics teacher educators' hybrid identities. ***Merrilyn Goos*** (Ireland) *and Margaret Marshman* (Australia).
[2] Analyzing challenges in the practice of a math teacher educator for developing community of math educators. ***Ruchi S. Kumar*** (India).
[3] Mathematics and science teacher educators learning induced by common research on professional vision. ***Nada Vondrova*** (Czech).
Session 2
[4] Teacher educators' preparation model: example from a successful professional development. ***Paola Sztajn,*** *Kristen Malzahn, and Reema Alnizami* (USA).
[5] Using a community of practice perspective to analyze mathematics teacher educator learning during lesson study. ***Melissa Soto****, Lara Dick, Mollie Appelgate, and Dittika Guptal* (USA).
[6] Characterizing mathematics teaching research specialists' mentoring in the context of Chinese lesson study. ***Zhenzhen He****, Feishi Gu, and Lingyuan Gu* (China).
[7] Didactical suitability criteria used by Italian teachers in lesson studies. ***Carola Manolino*** (Italy), *Viviane Hummes, Adriana Breda, Alicia Sánchez, and Vicenç Font* (Spain).
[8] The lesson study cultural transposition: from Chinese lesson study to Italian lesson study. ***Alessandro Ramploud****, Maria Mellone, Silvia Funghi, and Simone Esposito* (Italy).
[9] Using a nested structure of lesson study approach: a self-study as a mathematics teacher educator. ***Yinkang Wu*** (China).
[10] A collaborative work of four mathematics teacher educators. a study in Uruguay. ***Daniela Pages*** (Uruguay).
Session 3
[11] A collaborative self-study of two mathematics teacher educators learning and growing as culturally responsive pedagogues. ***Lindsay Keazer*** *and Kathleen Nolan* (Canada).
[12] Exploring power and oppression: a study of mathematics teacher educators' professional growth. ***Craig Joseph Willey****, Michael Richard Lolkus, Jill Newton, and Troy Bell* (USA).
[13] Differing contexts and tensions mathematics teacher educators experience in content courses for elementary preservice teachers. ***Hwa Young Lee****, Emily Miller, Travis Weilan, Tuyin An. And Daniel Clark* (USA).
[14] Developing mathematics education leaders in schools in Guatemala and implications for work in other countries. ***Chadd McGlone*** (USA).
[15] Transitioning between different identities: how the different positions assumed by the mathematics teacher educator impact their practice. ***Natalia Ruiz****, Nicole Fuenzalida, and Luz Valoyes-Chávez* (Chile).
[16] Mathematics teacher educators as role model: intentions and strategies. ***Helena Montenegro****, Salomé Martínez, and Francisco Rojas* (Chile).
[17] Experience of learning to teach mathematics: what do prospective teachers learn from their mathematics teacher educators? ***Francisco Rojas****, Helena Montenegro, and Flavio Guiñez* (Chile).
[18] Narratives of maths teachers: students & teacher ratio in mathematics classes in private schools. ***Sagar Dahal*** (Nepal).
Session 4
[19] Integrated mathematics teacher educators' professional development program. ***Haw-Yaw Shy****, Ting-Ying Wang, Yen-Ting Chen, Chi-Tai Chu, Chen-Ju Pai, and Mei-Hsien Chen* (Chinese Taipei).
[20] Talking across professional communities: teacher educator competencies in mathematics and in technology. ***Cengiz Alacaci*** (Norway)*., Bulent Cetinkaya, and Ayhan Kursat Erbas* (Norway).
[21] Mathematics teacher educators' knowledge for designing online professional development. ***Dinglei Huang*** (USA).
[22] Mathematics teachers' professional noticing in teaching of inverse functions and graphs in grade 12? ***Annie Mamoretsi Kgosi*** (South Africa).
[23] Examining teacher educator noticing during rehearsals of teaching: a focus on attending. ***Marta Kobiele*** (Canada).
[24] Mathematics teacher educator care and questioning in mathematics methods early field debriefing discussions. *Signe Kastberg, Lizhen Chen,* ***Sue Ellen Richardson****, and Mahtob Aqazade* (USA).
[25] Un/intelligent way to professional development of mathematics teachers: a case from Nepal. ***Amrit Bahadur Thapa*** (USA).

allows teachers to articulate their thoughts and ideas, further research is needed to identify the knowledge and practice of math teacher educators for developing the sense of the community. According to Ruiz et al.[15], the processes through which a novice facilitator navigates between teachers' and facilitators' identities (triggered by contextual aspects) is explored. The results indicate that to faithfully and flexibly replicate professional development programs in new and unfamiliar school contexts, it is critical to understand the processes of identification experienced by novice facilitators as part of their process of learning.

There are also several researches regarding collaborative work. Vondrova[3] reported the experience of a biology teacher educator and a mathematics teacher educator which started as collaborative research on professional vision but gradually became a learning experience for them. Pages' talk[10] was about a collaborative work of four MTEs in Uruguay. The researchers proposed them to plan a lesson in the first Calculus course of the mathematics teacher education program, to implement that lesson, and analyze it in a collective way. A theoretical model was found by using Classic Grounded Theory. The process called looking for agreements is resolved by the activation and eventual mobilization of the personal theories built in practice of each MTE, which constitutes the core category that emerged during the study. Keazer and Nolan[11] presented a collaborative self-study of two MTEs developing their own culturally responsive pedagogies (CRP) when teaching mathematics education courses. To answer the question "What do MTEs learn from attempts to grow and reflect on their own CRP?", they developed an MTE framework for growing CRP, which applied to their practice for data collection and for further iterations of examining their CRP.

MTEs play an important role in working with teachers. Paper by Sztajn et al.[4] described a model used to prepare MTEs to facilitate a professional development program that has demonstrated it can be implemented with integrity and has also shown positive impact on elementary teachers' knowledge and practice. In paper by Lee et al.[13], five MTEs who teach mathematics content courses for elementary preservice teachers (ePTs) at institutions across the USA present the differing contexts in which they teach such courses. The sequencing and integration of content and pedagogy, content coverage and mathematical rigor, and interactions with ePTs views and prior experiences in learning mathematics were explored from the perspective of MTEs. Montenegro et al.[16] reported a phenomenographic research which aimed to explore the approaches to modeling held by MTEs. Data were collected through semi-structured interviews conducted face-to-face with fifteen MTEs working in three Chilean primary initial teacher education programs. The analysis identified four approaches to modeling, ranging from performing pedagogical activities and interactions to developing teaching practices linked to the school classroom. Rojas et al.[17] reported on part a Self-Study aimed to investigate the challenges of two Chilean MTEs when teaching how to teach mathematics. The prospective teachers' perceptions of the teaching practices enacted by their MTEs was explored. Data were collected through focus groups and analyzed using thematic analysis. Conclusion showed prospective teachers look at the mathematics teacher educators as a role model and

would replicate some of their teaching practices when they become schoolteachers. The teacher educator facilitating the rehearsal plays a key role in supporting pre-service teachers' learning opportunities. Kobiele[23] presented an analysis of four teacher educators' noticing when facilitating rehearsals of one instructional activity — quick images. Using video-based interviews with each of the teacher educators, eight aspects within rehearsals that they attended to were identified. Kastberg et al.[24] examined how caring-relations influenced MTE questioning practice in the context of debriefing discussions with prospective teachers (PTs) during early-field experience linked to a mathematics methods course. Findings revealed that the MTE's ability to maintain focus on the PTs' objects of interest was informed by the MTE's feelings of reciprocal care.

Some contributions concerned the LESSON STUDY, a model of teacher education spread all over the world after the origin in the far east. Soto et al.[5] shared research on the use of lesson study for MTEs' professional development. Using Wenger's (1998) Social Theory of Learning framework based on communities of practice, they demonstrated how MTEs' learning changed across the process of the lesson study. He et al.[6] examined how mathematics teaching research specialists mentor practicing teachers during post-lesson debriefs of a lesson study in China. On the basis of the data analysis, a framework for analyzing mentoring activities emerged. The strengths and weaknesses of the teaching research specialists' mentoring strategies are identified through the framework, and suggestions to improve the teaching research specialists' mentoring strategies are discussed. Manolino et al.[7] aimed to identify the Didactical Suitability Criteria used by a group of Italian teachers participating in Lesson Studies. The written reflections and report documents — such as the Lesson Plans — are qualitatively analysed. The results suggest that all the Didactical Suitability Criteria are considered: the Epistemic, Cognitive, Interactional and Ecological criteria are particularly prominent; the Emotional and Mediational criteria sporadically appear. Ramploud et al.[8] aimed to show the process of deconstruction, functional to start a Cultural Transposition. Starting from the awareness of the "disorientation" generated by different cultural approaches to mathematics teaching, this process aims to produce versions of a didactic practice that are compatible with other cultural context and are suitable to support changes in teacher' beliefs. Wu[9] reported a nested structure of lesson study approach adopted in teaching pre-service mathematics teachers the course entitled Design of Mathematics Teaching. The underlying consideration and activities of each phase of lesson study at levels of MTE and preservice secondary teachers are both presented. Some preliminary findings regarding the effect of this approach on both the enrolled preservice secondary teachers and the MTE herself were provided.

Part of the researches concern the professional development of MTEs. Paper by Willey et al.[12] showcased the process and findings from an examination of MTEs professional growth as a result of engaging in a collaborative interrogation of critical texts outside of mathematics education. Findings suggest that this series of structured reading and dialogue led MTEs to develop a deeper understanding of the historical

movements and events that created todays local and global status quos. Shy et al.[19] reported a four-year (two stages) professional development program called "New Horizon of Mathematics (NHM)" in Chinese Taipei. The findings indicate that all MTEs and in-service teachers participating in the study have a great transformation on the understanding of CK and PCK and some satisfactory results on mathematics teacher education are obtained.

Other interesting research such as McGlone[14], reported that a program "Teachers2Teachers Global", starting in Guatemala, has developed and implemented teacher training strategies that are rooted in the best practices in mathematics education. The research problem of Dahal[18] was to know the practices of class size with student teacher ratio in private school in the context of Kathmandu district with academic performances in mathematics. Two maths teachers' stories were collected for knowing the gain and pain of the classroom with larger number of student and smaller number of student. Kgosi[22] discussed Mathematics Teachers' professional noticing in the teaching of inverse functions and graphs in Grade 12 in South Africa. As a result, the paper looks deeper in how professional noticing can be used to provide assistance for teachers to notice learner(s) mathematical thinking and how to interpret their mathematical understanding while learning inverse functions and graphs. Under the attention to the strong beliefs teachers possess of mathematical intelligence, Thapa's Paper[25] showed a case in Nepal using multi-paradigmatic research space to inquire about the dis/empowering environment teachers create in math class. The researcher discussed and shared the exploration and practices towards 'un/intelligent educational approach' for a reform in content, pedagogy and assessment of mathematics teaching through teacher development.

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